RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. SECOND SEMESTER EXAMINATION, MAY 2024
FIRST YEAR [BATCH 2023-25]

Date : 16/05/2024

MATHEMATICS

Time : 11.00 am – 1:00 pm

PAPER : CC 6

Full Marks : 50

Answer all, maximum you can score is 50.
All the symbols have their usual meanings.

1. Let $F=\mathbb{R}$ and $V=\mathbb{R}^2$ and T be the linear transformation form V to V which is rotation clockwise about the origin by $\frac{\pi}{2}$ radians. Show that V and O are the only $F[x]$ - submodules for T. [5]

2. Let M and N be two left modules over a commutative ring R. Prove that the set $\mathrm{Hom}_R(M,N)$ of all R-morphisms from M to N is again a left R-module. [6]

3. Let R be a commutative ring. Prove that $\mathrm{Hom}_R(R,R)$ and R are isomorphic as rings. [5]

4. Let N be a submodule of M. Prove that if both $\frac{M}{N}$ and N are finitely generated then so is M. [5]

5. Determine the Galois group of $(x^2-2)(x^2-3)(x^2-5)$. Determine all the subfields of the splitting field of this polynomial. [5+5]

6. Prove that $x^{p^n}-x+1$ is irreducible over $\mathbb{F}_p$ only when $n=1$ or $n=p=2$. [5]

7. Find a primitive generator for $\mathbb{Q}(\sqrt{2},\sqrt{3},\sqrt{5})$ over $\mathbb{Q}$. [6]

8. Determine the Galois group of the polynomial x^3-2 over $\mathbb{Q}$. [6]

9. Prove that α is algebraic over F if and only if the simple extension $\frac{F(\alpha)}{F}$ is finite. [6]

10. Prove that for n odd, $n>1$, $\Phi_{2n}(x)=\Phi_n(-x)$. [3]

11. Let θ be a root of $x^3+9x+6\in\mathbb{Q}[x]$. Find the inverse of $1+\theta$ in $\mathbb{Q}(\theta)$. [3]

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